1. **2 points** Disprove the following statements:
   
   1a. The sum of two numbers is always even.

   1b. The product of two distinct numbers of the form $6k + 5$ is also of the form $6k + 5$.

2. **4 points** Prove: If $a|b$ and $c \neq 0$, then $ac|bc$. 
3. **4 points** By considering three distinct cases, prove that if \( a \) is any integer, then \( a(a^2 + 2) \) is divisible by 3.

4. **4 points** Prove the following statement by induction: For all \( n \geq 1 \), \( 5|3^{n+1} + 2^{n+1} \).
5. **4 points** Prove that if \( \gcd(a, b) = 1 \), then \( \gcd(a, b^2) = 1 \).

6. **4 points** Find all solutions in positive integers of the diophantine equation \( 65x + 40y = 2020 \).
7. **4 points** Use mathematical induction to prove that

\[
1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2
\]

for all \( n \geq 1 \).

8. **4 points** Prove by direct computation. Given that the \( n \)th triangular number, \( T_n \), is given by

\[
T_n = \frac{n(n + 1)}{2}
\]

prove that the sum of two consecutive triangular numbers, \( T_n + T_{n+1} \), is always a perfect square.