MATH 302

Test 1
January 28, 2005

Name ____________________

Calculators may be used. Put your answers in the space provided circle your answers. The maximum score on the test is 30 points.

1. [2 points] Disprove the following statements:

1a. The sum of two numbers is always even.

1b. The product of two distinct numbers of the form $6k + 5$ is also of the form $6k + 5$.

2. [4 points] Prove: If $a|b$ and $c \neq 0$, then $ac|bc$.

3. [4 points] By considering three distinct cases, prove that if a is any integer, then $a(a^2+2)$ is divisible by 3.

4. [4 points] Prove the following statement by induction: For all $n \geq 1$, $5 | 3^{3n+1} + 2^{n+1}$.

5. 4 points Prove that if $\gcd(a, b) = 1$, then $\gcd(a, b^2) = 1$.

6. 4 points Find all solutions in positive integers of the diophantine equation $\boxed{65x + 40y = 2020}$.

7. 4 points Use mathematical induction to prove that

$$1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2$$

for all $n \geq 1$.

8. 4 points Prove by direct computation. Given that the nth triangular number, T_n, is given by $T_n = \dfrac{n(n+1)}{2}$, prove that the sum of two consecutive triangular numbers, $T_n + T_{n+1}$, is always a perfect square.