

MATH 302

TEST 3
MARCH 4, 2005

NAME _____

Put your answers in the space provided. Show your reasoning. Calculators are to be used when appropriate. The maximum score on the test is 30 points.

1. 4 points Prove that $\phi(n^3) = n^2\phi(n)$.

2. 4 points Find four solutions of $\phi(n) = 18$.

3. 4 points Find all solutions of $\sigma(n) = 18$.

4. 4 points Show that if $2^k - 1$ is a prime, then $n = 2^{k-1}(2^k - 1)$ satisfies the equation $\sigma(n) = 2n$.

5. 4 points Show that if $\gcd(m, n) = 2$, then $\phi(mn) = 2\phi(m)\phi(n)$

6. 4 points For all n characterize $\phi(6n)$ in terms of $\phi(n)$.

7a. 1 point Compute $\tau(420)$

Answer _____

7b. 1 point Compute $\sigma(420)$

Answer _____

7c. 1 point Compute $\phi(420)$

Answer _____

8. 3 points Find the units digit of 3^{102} by means of Euler's Theorem.

Answer _____