MATH 302

Assignment 1
January 10, 2003

The following problems are to be done using Mathematical Induction. They are not to be turned in but to be presented on the board. Feel free to work together.

1. Prove: $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \dfrac{n(n+1)(n+2)}{3}$.

2. Prove: $\displaystyle\sum_{k=1}^{n} (2k-1) = n^2$.

3. Prove: $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$.

4. Prove: $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.

5. Use the fact that $|x+y| \leq |x| + |y|$ for all real numbers to prove that

   $$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|$$

   for all real numbers $x_1, x_2, \ldots x_n$.

6. Use induction to prove that the product of two consecutive integers is always divisible by 2.

7. Use induction to prove that the product of three consecutive integers is always divisible by 6.

8. Using induction prove that $\dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \cdots + \dfrac{n}{2^n} = 2 - \dfrac{n+2}{2^n}$.

9. Using induction prove that $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots + \dfrac{1}{n^2} \leq 2 - \dfrac{1}{n}$.

10. Establish the Bernoulli inequality: If $1 + a > 0$, then

    $$(1+a)^n \geq 1 + na$$

    for all $n \geq 1$.