MATH 304

Test 2
February 11, 2005

Name ____________________________

Put your answers in the space provided. Show your reasoning. The maximum score on the test is 30 points. Calculators may be used unless specifically restricted.

1. 3 points Find an orthogonal basis for $\mathbf{R}^3$ which contains the vector $[1,2,3]^T$ and uses no fractions. Circle your answer.

2. 3 points Consider the following three vectors in $\mathbf{R}^4$: $\mathbf{a}_1 = [1,-1,0,0]^T$, $\mathbf{a}_2 = [1,0,-1,0]^T$, $\mathbf{a}_3 = [1,-1,-1,1]^T$. Use the Gram-Schmidt process to construct an orthogonal basis with no fractions for Span($\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$). Circle your answer.

3. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 4 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$ A and B have the same column spaces and let us call it W.

3a. [2 points] Find the vector in W which is closest to $\mathbf{v} = [2, 1, 2, 2]^T$.

Answer ______________________

3b. [4 points] Without using the QR feature on your calculator find the QR factorization of the matrix A. **Circle your answers**.

4. [2 points] If $\mathbf{x}$ is in the null space of a matrix $A^T A$, prove that $\mathbf{x}$ is in the null space of A.

5. 3 points Prove that if B is similar to a non-singular matrix A, then B^{-1} is similar to A^{-1}.

6. 3 points Let W be the subspace of $\mathbf{R}^4$ spanned by $[1,2,1,2]^T$ and $[0,1,0,1]^T$. Find a basis for $W^\perp$. **Circle your answer**.

7. 2 points Give the definition of an *orthogonal matrix*.

8. 4 points Consider the following vectors in $\mathbf{R}^3$: $\mathbf{u} = [3, 2, -1]^T$ and $\mathbf{v} = [-2, 2, 5]^T$. Let W be Span($\mathbf{u}$). Find the projection of $\mathbf{v}$ onto the orthogonal complement of W.

Answer ______________________________

9. 4 points Find the equation of the line, $y = b + mx$, which best fits the three points $(1, 2)$, $(2, -3)$ and $(5, -5)$.

Answer ______________________________