

# MATH 304

TEST 3  
MARCH 1, 2005

NAME \_\_\_\_\_

Put your answers in the space provided. Show your reasoning. The maximum score on the test is 30 points.

Calculators may be used unless specifically restricted.

1. 3 points Let  $\mathbf{u} = [1/3, -2/3, 2/3]^T$ . Show that  $\mathbf{u}$  is an eigenvector for  $\mathbf{u}\mathbf{u}^T$  and find all of its eigenvalues. Do not use your calculator.

Answer \_\_\_\_\_

2. 4 points Let  $\mathbf{u}$  be a unit vector in  $R^n$ . Let  $Q = I - 2\mathbf{u}\mathbf{u}^T$ . Show that  $Q$  will always be an orthogonal matrix.

3. 5 points Find a singular value decomposition, SVD, for the matrix  $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$ . Circle your answer.

4. 4 points Let  $W$  be the subspace of  $\mathbf{R}^4$  spanned by  $[1, 2, 1, 2]^T$  and  $[0, 1, 0, 1]^T$ . Find a basis for  $W^\perp$ . Circle your answer.

5. 4 points If  $A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ , then its SVD is

$$\begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} \sqrt{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{bmatrix}$$

Find, circle and label orthonormal basis for the row space of  $A$ , nullspace of  $A$ , column space of  $A$ , and nullspace of  $A^T$ .

6. Consider the quadratic form  $Q(\mathbf{x}) = 2x_1^2 + 3x_2^2 + 2x_3^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3$ .

6a. 3 points Find the maximum value of  $Q(\mathbf{x})$  subject to the constraint  $\|\mathbf{x}\| = 1$ .

Answer \_\_\_\_\_

6b. 3 points Find a unit vector  $\mathbf{u}$  where this maximum is attained.

Answer \_\_\_\_\_

7. 4 points Find  $P$  and the new quadratic form when one makes the change of variable,  $\mathbf{x} = P\mathbf{y}$  that transforms the quadratic form  $Q(\mathbf{x}) = x_1^2 - x_2^2 + 4x_1x_3 - 4x_2x_3$  into a quadratic form with no cross-product term. CIRCLE YOUR ANSWER.