1. **3 points** Let $\mathbf{u} = \begin{bmatrix} 1/3, -2/3, 2/3 \end{bmatrix}^T$. Show that $\mathbf{u}$ is an eigenvector for $\mathbf{u}\mathbf{u}^T$ and find all of its eigenvalues. Do not use your calculator.

Answer ____________________________

2. **4 points** Let $\mathbf{u}$ be a unit vector in $\mathbb{R}^n$. Let $Q = I - 2\mathbf{u}\mathbf{u}^T$. Show that $Q$ will always be an orthogonal matrix.
3. 5 points Find a singular value decomposition, SVD, for the matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$. Circle your answer.
4. \[4 \text{ points}\] Let \( W \) be the subspace of \( \mathbb{R}^4 \) spanned by \([1, 2, 1, 2]^T\) and \([0, 1, 0, 1]^T\). Find a basis for \( W^\perp \). Circle your answer.

5. \[4 \text{ points}\] If \( A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \), then its SVD is

\[
\begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} \sqrt{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{bmatrix}
\]

Find, circle and label orthonormal basis for the row space of \( A \), nullspace of \( A \), column space of \( A \), and nullspace of \( A^T \).
6. Consider the quadratic form \( Q(x) = 2x_1^2 + 3x_2^2 + 2x_3^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3. \)

6a. 3 points Find the maximum value of \( Q(x) \) subject to the constraint \( \|x\| = 1. \)

Answer _____________________________

6b. 3 points Find a unit vector \( u \) where this maximum is attained.

Answer _____________________________

7. 4 points Find \( P \) and the new quadratic form when one makes the change of variable, \( x = Py \) that transforms the quadratic form \( Q(x) = x_1^2 - x_2^2 + 4x_1x_3 - 4x_2x_3 \) into a quadratic form with no cross-product term. Circle your answer.