MATH 304

Quiz 2 Solutions
February 6, 2007

Name ____________________

Put your answers in the space provided. Show your reasoning. Calculators may not be used. The maximum score on this quiz is 6 points.

1. 4 points Consider the following two vectors in $\mathbf{R}^3$: $\mathbf{u} = [-2, 1, 3]^T$ and $\mathbf{v} = [3, 3, 4]^T$.

1a. Prove that $\mathbf{u}$ is shorter than $\mathbf{v}$.

$||\mathbf{u}||^2 = 4 + 1 + 9 = 14, \quad ||\mathbf{v}||^2 = 9 + 9 + 16 = 34$. Therefore $\mathbf{u}$ is shorter than $\mathbf{v}$

1b. Compute the distance from $\mathbf{u}$ to $\mathbf{v}$

The distance from $\mathbf{u}$ to $\mathbf{v}$ is $||\mathbf{v}-\mathbf{u}|| = \sqrt{(3-(-2))^2 + (3-1)^2 + (4-3)^2} = \sqrt{25+4+1} = \boxed{\sqrt{30}}$

1c. Find an unit vector in the direction opposite to $\mathbf{u}$

From 1a. the length of $\mathbf{u}$ is $\sqrt{14}$. The answer is $\boxed{\dfrac{-1}{\sqrt{14}}[-2, 1, 3]^T}$

1d. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

The projection is $\dfrac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v} = \dfrac{-6+3+12}{9+9+16}\mathbf{v} = \boxed{\dfrac{9}{34}[3, 3, 4]^T}$

1e. Find a non-zero vector which is orthogonal to $\mathbf{u}$

Two possible answers are $\boxed{[-1, 1, -1]^T \text{ or } [0, 3, -1]^T}$

1f. Find $\mathbf{u}\mathbf{v}^T$

$$\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} [3, 3, 4] = \boxed{\begin{bmatrix} -6 & -6 & -8 \\ 3 & 3 & 4 \\ 9 & 9 & 12 \end{bmatrix}}$$

2. 2 points Let W be the subspace of $\mathbf{R}^4$ spanned by $\mathbf{u}_1 = [1, 1, 0, 0]^T$ and $\mathbf{u}_2 = [1, -1, 0, 0]^T$. Find a basis for $W^\perp$. Circle your answer.

There are many answers. One is $\boxed{\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}}$