1. Write a quadratic form in the three variables $x$, $y$ and $z$ which has exactly one cross term.
   Many answers are possible, e.g. $Q(x, y, z) = 4x^2 + xy - z^2$

2. Find all eigenvalues for the singular matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$. Also find corresponding eigenvectors. Circle your answers.

   Since $A$ is singular, $\lambda_1 = 0$ is an eigenvalue. The constant row sum is 6 so $\lambda_2 = 6$. The trace is 4 so the third eigenvalue is $\lambda_3 = -2$

   $\lambda_1 = 0. \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

   So an eigenvector is $x_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

   Since 6 is the row sum an eigenvector is $x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   The eigenvectors corresponding to $\lambda_3 = -2$ will be orthogonal to the above and so by inspection one is $x_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

3. Find the matrix for the quadratic form $2x^2 + 3y^2 + 5z^2 - xy + 6xz$

   Circle your answer.

   The matrix is $\begin{bmatrix} 2 & -\frac{1}{2} & 3 \\ -\frac{1}{2} & 3 & 0 \\ 3 & 0 & 5 \end{bmatrix}$