

Put your answers in the space provided. Show your reasoning. Calculators may not be used. The maximum score on this quiz is 6 points.

1. 1 point Write a quadratic form in the three variables x , y and z which has exactly one cross term.

Many answers are possible, e.g. $Q(x, y, z) = 4x^2 + xy - z^2$

2. 3 points Find all eigenvalues for the singular matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$. Also find corresponding eigenvectors. Circle your answers.

Since A is singular, $\lambda_1 = 0$ is an eigenvalue. The constant row sum is 6 so $\lambda_2 = 6$. The trace is 4 so the third eigenvalue is $\lambda_3 = -2$

$$\lambda_1 = 0. \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

So an eigenvector is $\mathbf{x}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Since 6 is the row sum an eigenvector is $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

The eigenvectors corresponding to $\lambda_3 = -2$ will be orthogonal to the above and so by inspection

one is $\mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

3. 2 points Find the matrix for the quadratic form

$$2x^2 + 3y^2 + 5z^2 - xy + 6xz$$

Circle your answer.

The matrix is $\begin{bmatrix} 2 & -\frac{1}{2} & 3 \\ -\frac{1}{2} & 3 & 0 \\ 3 & 0 & 5 \end{bmatrix}$